



## COURSE DESCRIPTION CARD - SYLLABUS

Course name

Modeling of discrete and continuous systems [S2MiBM2>MUDiC1]

### Course

Field of study

Mechanical Engineering

Year/Semester

1/1

Area of study (specialization)

–

Profile of study

general academic

Level of study

second-cycle

Course offered in

Polish

Form of study

full-time

Requirements

compulsory

### Number of hours

Lecture

30

Laboratory classes

15

Other

0

Tutorials

0

Projects/seminars

0

### Number of credit points

3,00

### Coordinators

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### Lecturers

### Prerequisites

1. Basic knowledge of mechanics and mathematics corresponding to the core curriculum for the first-cycle studies. 2. The ability to solve elementary problems in mechanics to the extent consistent with the core curriculum of the first-cycle studies. 3. The ability to independently study and develop skills using the indicated literature, Internet sources and any other available opportunities. 4. Awareness of the need to constantly learn and improve one's skills.

### Course objective

1. Getting to know and understanding the basic concepts, laws, and equations of analytical mechanics used to create a mathematical model of linear and nonlinear mechanical discrete systems. 2. Developing the ability to create a physical model and a mathematical model of discrete mechanical systems. Developing the ability to solve problems of motion and equilibrium of complex mechanical systems and the ability to analyze the obtained solutions.

### Course-related learning outcomes

Knowledge:

1. A student who has completed the course knows and is able to explain the basic concepts of analytical

mechanics regarding discrete mechanical systems with constraints.

2. The student is able to write and explain Lagrange equations of the second kind, the principle of virtual work and the Dirichlet principle and consciously consider the conditions of their applicability.

3. The student knows the basic concepts of the dynamics of discrete systems.

Skills:

1. The student is able to create a discrete physical model of mechanical systems being parts engineering devices and structures.

2. Using Lagrange equations of the second type, he can derive equations of motion and formulate the conditions necessary to create a mathematical model of the problem under consideration.

3. Using advanced programs for symbolic and numerical calculations, he can obtain the solution to the given problem.

4. Using knowledge of the typical behavior of discrete systems, he can analyze the obtained solution.

Social competences:

1. The student understands the importance of knowledge in the modern world. He is also well aware that the rapid development of knowledge causes the need for lifelong learning.

2. He can organise the processes of learning and self-education.

### Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Lectures: Completion of the course is in written form and consists of two parts: theoretical and practical.

Grades:

5.0 - if the ratio of sums of achieved and total points is bigger than 90,1%;

4.5 - if the ratio of sums of achieved and total points is between 80,1-90%;

4.0 - if the ratio of sums of achieved and total points is between 70,1-80%;

3.5 - if the ratio of sums of achieved and total points is between 60,1-70%;

3.0 - if the ratio of sums of achieved and total points is between 50,1-60%;

2.0 - if the sum is smaller than 50%.

Laboratory classes:

assessment of written studies prepared by students based on tasks completed during classes.

The required elements of the study are:

- Mathematica's script containing the creation of a mathematical model, solution of the model equations, and presentation of the obtained results in graphical form,

- analysis of the system's behaviour based on the obtained results.

The knowledge necessary to complete the task and the ability to solve the problem are assessed.

Assessment on a point scale. The condition for passing is obtaining at least 50% of the total number of points; the grading scale is linear.

### Programme content

Lectures:

Discrete and continuous mechanical systems. Modelling in mechanics. Physical and mathematical models. Model verification.

Free and constrained mechanical systems. Motion constraints and their classification. Structural analysis of complex discrete mechanical systems.

Generalised coordinates and generalised velocities. Generalised forces. Lagrange's equations of the second kind.

Modelling of elastic interactions. Modelling of resistance forces in the motion of mechanical systems. The Rayleigh dissipation function. Friction force models.

Principle of virtual work.

Dirichlet's principle.

Analysis of the motion of a system with one and two degrees of freedom.

Elements of analysis of vibrations of linear and nonlinear systems.

Selected problems of dynamics of discrete nonlinear systems. Phase plane.

Laboratory classes:

Introduction to Mathematica software.  
Numerical solution of initial problems.  
Kinetic energy of complex mechanical systems.  
Lagrange equations of the second kind applied to systems with one and two degrees of freedom.  
Planetary gears.  
Analysis of the motion of nonlinear systems in phase space.

## Course topics

Lectures:

Introduction. Discrete and continuous mechanical systems. The essence of modeling. Mathematical models of discrete and continuous mechanical systems. Characteristics of methods for solving equations of the mathematical model. Modelling steps. Model verification. Physical model. Characteristics of the most commonly used model assumptions.

Basic elements of discrete mechanical systems. Laws of motion - Newton-Euler formulation. Inertial properties of a rigid body. Kinetic energy of elements of a discrete mechanical system.

Lagrange's formalism. Free and constrained discrete systems. Positions and velocities. Motion constraints and their nature. Constraint function. Classification of constraints. Holonomic constraints. Degrees of freedom of discrete systems under the action of bilateral holonomic constraints. Structural analysis of complex mechanical systems. Examples. Unilateral constraints and degrees of freedom of a system.

Gradient of the constraint function at a point. Conditions imposed on the velocities of system points resulting from non-stationary geometric bilateral constraints. Possible positions and velocities. Virtual displacements. Generalised coordinates. Generalised velocities. Virtual displacement expressed by generalised coordinates. Virtual work. Ideal constraints. Generalised forces.

Potential and conservative forces. Uniform field of gravity as a model of gravitational forces. Elastic force in the linear range. Potential energy of gravity and elastic force. Generalized forces derived from conservative forces and their relationship with the potential energy. Solving problems.

The first form of the Lagrange equations of the second kind. Solving problems. The Lagrange function and the second form of the Lagrange equations of the second kind. Solving problems. The special form of the Lagrange equations of the second kind for conservative mechanical systems.

Rolling without slipping. Application of Lagrange's equations to the analysis of the motion of planetary gears. Solving problems.

Modelling of resistance forces. Motion of a mass point in a medium with resistance proportional to the square of velocity - solution of the problem using Lagrange's equations of the second kind. Viscous resistance. Rayleigh's function. Friction force in dynamic problems. Solving problems.

Principle of virtual work. Three formulations of the principle of virtual work. Principle of virtual work for systems with ideal constraints. Solving problems. Application of the principle of virtual work to determining the reaction forces of hinged beams.

Necessary and sufficient conditions for the equilibrium of a system of conservative forces. Equilibrium positions of a system of conservative forces. Stationary points of potential energy. Types of equilibrium positions of a system of conservative forces. Dirichlet's principle. Sufficient condition for stable equilibrium. Determination of equilibrium positions and assessment of their stability - solving problems. Application of Dirichlet's principle to determining initial conditions. Solving tasks.

Rods, shafts and beams as elastic elements in discrete systems. Equivalent stiffness of a system of elastic elements. Reduced mass of elastic elements.

Phase space and phase plane - basic concepts. Analysis of the motion of an autonomous system with one degree of freedom in the phase plane. Properties of phase trajectories.

Vibrations of single-degree-of-freedom systems - introduction. Free linear vibrations of a single-degree-of-freedom system. Harmonic oscillator.

Modelling of restitutive forces in nonlinear systems. Nonlinearities of geometric and physical nature. Nonlinear relations for resistance forces. Internal friction. Basic models of friction force. Amplitude-frequency interaction in anharmonic oscillators. Phase portraits of selected anharmonic oscillators. Forced vibrations and resonance in nonlinear systems.

Laboratory classes:

Short introduction to Mathematica program with elements of vector calculus.

Solving and analysing the solution of the initial problem. Graphical presentation of results.

Structural analysis of the complex mechanical systems. Determination of the number of degrees of freedom.

Kinetic energy of complex mechanical discrete systems. Koenig's theorem.

Structural analysis of complex mechanical systems. Determination of degrees of freedom.

Lagrange equations of the second kind. Equations of motion of discrete systems with one degree of freedom.

Lagrange equations of the second kind. Equations of motion of discrete systems with two degrees of freedom. Rayleigh's function.

Modelling and analysis of planetary gear motion.

Principle of virtual work.

Dirichlet's principle.

Analysis of the motion of an autonomous system on the phase plane.

Determination of the vibration period - amplitude relationship for a nonlinear system.

## Teaching methods

Lectures: lecture aided by multimedia presentations, solving tasks and discussion of the obtained results.

Laboratory classes:

- creating mathematical models of selected systems,
- solving problems using the Mathematica program,
- presenting results in the form of graphs and animations,
- analysing the results.

An online course is available on the e-Kursy platform, supporting and supplementing the teaching process carried out during lectures and laboratories.

## Bibliography

Basic:

1. J. Grabski, J. Strzałko, B. Mianowski, Podstawy mechaniki analitycznej, Wydawnictwo Politechniki Łódzkiej, Łódź 2016.

2. J. R. Taylor, Mechanika klasyczna, t. 2, PWN, Warszawa, 2006.
3. Z. Gutowski, Mechanika analityczna, PWN.

Additional:

1. W. Rubinowicz, W. Królikowski, Mechanika teoretyczna, PWN.
2. G.K. Susłow, Mechanika teoretyczna, PWN.

### Breakdown of average student's workload

|   | Hours | ECTS |
|---|-------|------|
| Total workload  | 75    | 3,00 |
| Classes requiring direct contact with the teacher   | 45    | 2,00 |
| Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation) | 30    | 1,00 |